

PIRMAP: Efficient Private Information Retrieval for MapReduce

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Abstract. Private Information Retrieval (PIR) allows a user to retrieve bits from a database while hiding the user’s access pattern. However, the practicality of PIR in a real-world cloud computing setting has recently been questioned. In such a setting, PIR’s enormous computation and communication overhead is expected to outweigh the cost saving advantages of cloud computing. In this paper, we first examine existing PIR protocols, analyzing their efficiency and practicality in realistic cloud settings. We identify shortcomings and, subsequently, present an efficient protocol (PIRMAP) that is particularly suited to MapReduce, a widely used cloud computing paradigm. PIRMAP focuses especially on the retrieval of large files from the cloud, where it achieves good communication complexity with query times significantly faster than previous schemes. To achieve this, PIRMAP enhance related work to allow for optimal parallel computation during the “Map” phase of MapReduce, and homomorphic aggregation in the “Reduce” phase. To improve computational cost, we also employ a new, faster “somewhat homomorphic” encryption, making our scheme practical for databases of useful size while still keeping communication costs low. PIRMAP has been implemented and tested in Amazon’s public cloud with database sizes of up to 1 TByte. Our evaluation shows that non-trivial PIR such as PIRMAP can be more than one order of magnitude cheaper and faster than trivial PIR in the real-world.

Keywords: Privacy, MapReduce, cloud computing, Private Information Retrieval

1 Introduction

Cloud computing has recently been identified as an important business technology [10], as it offers greater flexibility and reduced costs to companies outsourcing data and computation. The cost advantage of cloud computing comes from the fact that cloud users do not need to maintain their own, expensive data center, but instead can pay a cloud provider for hosting. However, despite the hype, hesitations remain among potential cloud users. Lack of security and privacy guarantees has been identified as a major adoption obstacle for both large enterprise [19] and governmental organizations [14].

Privacy issues stem from the fact that, when a user hands over his information to the cloud, control of that data is relinquished to the cloud. Public clouds are threatened by hackers due to the much larger target they present. Insiders such as data center administrative staff can easily access private data that has been outsourced. Such attacks

are realistic and have already been reported [11, 21]. This shows that users need to take precautions to ensure the privacy and integrity of their cloud data.

While encryption of data at rest helps to protect data confidentiality, it is often not sufficient: the subsequent data access patterns can leak information about patients. For example, if the outsourced data in question contains encrypted patient records, the cloud provider might learn that a patient has been diagnosed with cancer when it sees that the patient's records have been retrieved by an oncologist. In general, it is hard to assess which information may be leaked, what inferences an adversary may make, and what risks the leakage may incur.

Private Information Retrieval (PIR) offers a solution to the information leakage problem by hiding access patterns [6, 12], independent of an encryption mechanism. In PIR, a database stored at a server holds n strings each of size l bits, and a user can query for one l bit string without leaking *which* string to the database. A trivial way to accomplish this is to simply transmit the entire database to the user. However, this is *communication* inefficient, and research focuses on achieving lower communication bounds. In contrast, *computational* cost of every PIR scheme is necessarily $O(n \cdot l)$, as the server must "touch" every piece of the database if the server is to remain oblivious of the requested piece.

In existing work, the server's computation is comprised of expensive cryptographic operations over the entire database. Because of the significant overhead this imposes, it has recently been questioned whether PIR will ever become *practical* in a real-world cloud computing setting, cf. Sion and Carbunar [18]. Typically, cloud providers such as Amazon charge their customers for both data transfer *and* CPU hours [2]. Due to the necessary condition that PIR protocols compute over the entire database for each query, it has been argued that trivial PIR (retrieving the whole $(l \cdot n)$ bit database) is not only faster, but also *cheaper* for the cloud customer compared to a PIR query that involves lengthy computation [5, 16, 18].

Another open question is how to perform PIR in a real-world cloud computing environment. In contrast to a single machine server storing the data, one of the biggest challenges in cloud computing is a design that scales easily to the large distributed systems which are characteristic in cloud settings. In order to alleviate this difficulty, major cloud providers (e.g., Amazon, Google, IBM, Microsoft) offer an interface to the prominent MapReduce [8] API for distributed computing to their users. MapReduce comprises not only parallelization ("Map") of work, but an aggregation ("Reduce") of individual results to keep computational burden on the user side low.

This paper considers the single-server, computationally-private information retrieval setting. This is appropriate, because, although a cloud provider may allow access to many servers, they must all be considered a single "trust entity": they are under the control of the same organization. Trust-wise, the cloud as a single, large server with many distributed CPUs. In the single-server setting, it is known that unconditionally secure PIR cannot be more efficient than transferring the entire database [7], so we are concerned instead with a computationally secure protocol (cPIR). Further use of the term PIR will be in reference to computationally-secure PIR, unless otherwise noted.

We present PIRMAP, an efficient single-server cPIR protocol suited for MapReduce clouds. PIRMAP especially targets retrieval of relatively large files, a more specific

setting than considered by previous work. In a scenario with n files each of size l bits and $l \gg n$, PIRMAP achieves communication complexity linear in l with low constants. PIRMAP is designed for and leverages MapReduce parallelization and aggregation. We have implemented PIRMAP in Hadoop MapReduce, and its performance will be presented in Section 5.

Our contributions are:

1.) An analysis of existing work with respect to its practicality in a MapReduce cloud setting.

2.) PIRMAP, an efficient, practical PIR scheme for MapReduce with optimal communication complexity $O(l)$ when retrieving an l bit file for $l \gg n$. Additionally, PIRMAP has computational complexity as good or better than prominent related work due to use of a more efficient homomorphic encryption. PIRMAP runs on top of standard MapReduce, not requiring changes to the underlying cloud infrastructure.

3.) An implementation of PIRMAP that is usable in real-world MapReduce clouds today, e.g., Amazon. We evaluate PIRMAP, first, in our own (tiny) local cloud and, second, with Amazon’s cloud. We verify its practicality by scaling up to 1 TByte of data in Amazon’s cloud setting. Compared to the previously largest single database PIR experiment with up to 28 GByte of data [16], this demonstrates the efficiency and practicality of PIRMAP in the real-world. PIRMAP is more than one order of magnitude *cheaper* and *faster* than trivial PIR, and – in our particular scenario – we can significantly outperform related work. PIRMAP’s source code is available for download [17].

2 Problem Statement

PIR: A PIR protocol is a series of interactions between a user and a server storing n files that results in the user retrieving one file while the server does not learn which file. More formally, the server cannot guess with probability greater than $1/n$ which file was queried. Note that this probability does not increase over multiple queries. This effectively hides the user’s access pattern as each query is computationally indistinguishable from the others. The only information leaked is the number of files queried.

A $CPIR_l^n$ protocol (**Query**, **Transfer**, **Recover**) is a computationally-secure private information retrieval protocol over a database of n elements, each of bit length l . The **Query** function generates the query and sends it to the server. **Transfer** is run on the server and involves transforming the received query into the results of the query and sending it back to the client. Finally, **Recover** is run by the client to transform the response from the server into the correct plaintext query result.

MapReduce: With the trend towards more computers and more cores rather than faster individual processors, it is important that any practical PIR implementation be deployable in a way that can take full advantage of parallel and cluster computing environments. Perhaps the most widely adopted architecture for scaling parallel computation in public clouds today is Google’s MapReduce [8]. Its design allows for a set of computations, or “job”, to be deployed across many nodes in a cloud cloud. Its biggest advantage is that it scales transparently to the programmer. That is, once an implementation is written using MapReduce, it can run on any number of nodes in the data center, from one up to hundreds or thousands, without changes in the code. This is managed

by splitting computation into two phases, each of which can be run in parallel on many computing nodes.

The first phase is called the “Map” phase. MapReduce will automatically split the input to the computation equally among available nodes in the cloud data center, and each node will then run a function called *map* on their respective pieces (called *Input-Splits*). It is important to note that the splitting actually occurs when the data is uploaded into the cloud (in our case when the patient record/files are uploaded) and not when the job is run. This means that each “mapper” node will have local access to its *InputSplit* as soon as computation has started and you avoid a lengthy copying and distributing period. The map function runs a user defined computation on each *InputSplit* and outputs a number of *key-value* pairs that go into the next phase.

The second phase, “Reduce”, takes as input all of the key-value pairs emitted by the mappers and sends them “reducer” nodes in the data center. Specifically, each reducer node receives a single key, along with the sequence of values output by the mappers which share that key. The reducers then take each set and combine it in some way, outputting a single value for each key.

Despite being widely used, MapReduce is a very specific computational model and not all algorithms can be easily adapted to it. A practical PIR such as PIRMAP has to take its specifics into account – as we will see below.

3 Related Work Analysis

Despite the large amount of theoretical research, there has not been much investigation into practical PIR. Recently, Trostle and Parrish [20] experimented with the state of the art and found that it took at least 8 minutes to retrieve a file of size only 3 MB, out of a data set of 3 GB on commodity hardware. Additionally, existing schemes have significant communication overhead for files of that size and larger. For example, Lipmaa [13], enjoying the most efficient asymptotic communication complexity today, requires 30 MB of communication in the above scenario.

In examining related work, we make two nonstandard observations which apply to our MapReduce cloud setting. First, since computation for PIR must necessarily be $O(n \cdot l)$, computation will most likely be the bottleneck in any protocol. The time to communicate queries and responses will be relatively small in our setting, compared to the time it takes to calculate over the entire database. Therefore, it may be useful to trade some extra communication for smaller computational burden. The second observation relates to MapReduce itself. Since each mapper may run on a different node in the cloud, communication and synchronization within the cloud are very expensive. To take full advantage of the cloud, there should be few, if any, inter-dependencies between stages of computation.

Overview One of the first cPIR schemes to achieve sublinear communication is that of Kushilevitz and Ostrovsky [12], using an additively homomorphic cipher \mathcal{E} as follows:

1. The server arranges an n elements database as a $\sqrt{n} \times \sqrt{n}$ matrix
2. *Query*(x) – The user generates a vector v of length \sqrt{n} where $v_x = \mathcal{E}(1)$, and $v_i = \mathcal{E}(0), \forall i \neq x$

3. **Transfer**(v) – Vector v is sent to the server. The server multiplies v with the database matrix and returns the result as vector v'
4. **Recover**(v') – v' now consists of \sqrt{n} encrypted elements, one of which is the element the user is interested in. The user chooses this element and decrypts it, discarding the rest.

The scheme is sound, because the user sends a vector v which “zeroes out” all the rows in the matrix besides the row requested. The communication costs is $O(l \cdot \sqrt{n})$ which is still quite high for large values of n . Kushilevitz and Ostrovsky [12] also show that this protocol can be repeated recursively to achieve communication less than $O(n^c)$ for any $c > 0$, but at the cost of worse constants and computational complexity. This “square-root solution” computes only once over the database, but has impractically large communication costs.

Lipmaa [13] reduces communication complexity to $O(l \cdot \log n + k \cdot \log^2 n)$, where k is a security parameter. This is accomplished by generalizing the Kushilevitz and Ostrovsky [12] scheme into a family of protocols, parameterized by a dimension α . The Kushilevitz and Ostrovsky [12] scheme can be seen as two-dimensional, because the database elements are arranged in a $\sqrt{n} \times \sqrt{n}$ matrix. If $\alpha = \log(n)$, the database is viewed as a $\log(n)$ dimensional $2 \times 2 \times \dots \times 2$ matrix. In this manner, the user may send a sequence of $\log(n)$ vectors, each only length 2 which “zero out” half of the remaining database elements at each step. It is conceptually similar to specifying a path in a binary tree, and when the leaf node is reached the only element that will remain non-zero is the one corresponding to that node.

The down side of this scheme is that it requires computing over the entire database twice ($\sum_{i=1}^{\log n} \frac{1}{2^i} = 2$). While not a significant problem asymptotically, as mentioned before, cPIR schemes are usually bottlenecked by their computational cost. Increasing this cost by 100% dramatically effects the efficiency of the scheme, especially given the cloud setting where twice the computation corresponds to twice the monetary cost. Another problem is that the computational complexity of Lipmaa [13] is $O(n \cdot l \cdot k^2)$, because it requires modular exponentiations on ciphertexts of size k . This is a significant overhead that we can avoid by using a more efficient homomorphism.

Although Lipmaa [13] requires only one round of communication with the user, it is iterative in that it requires $\log(n)$ rounds of computation on the server, each round depending on the output of the previous round and operating on a smaller set of data. As we will see, a large part of the cost for MapReduce is in initializing the parallel computation, so restarting MapReduce $\log(n)$ times adds a significant overhead.

More recently, Aguilar-Melamine and Gaborit [1] have proposed a scheme using lattice-based homomorphic encryption (similar to NTRU) which has much better computational complexity, both asymptotically and in practice. Their scheme also takes advantage of the fact that modern GPUs can solve large parallel problems very quickly. Aguilar-Melamine and Gaborit [1] are able to achieve fast query response times, but at the cost of a large communication cost. Additionally, their protocol is tuned specifically to the requirements of GPUs and would not scale well in a real-world distributed cloud environment like MapReduce.

Using a different encryption scheme based on the Trapdoor Group Assumption, Trostle and Parrish [20] propose a more traditional PIR scheme based on Kushilevitz

Table 1: Communication and computational complexities of related work.

	Communication	Computation
Kushilevitz and Ostrovsky [12]	$O(\sqrt{n} \cdot l)$	$O(l \cdot n \cdot k^2)$
Lipmaa [13]	$O(l \cdot \log(n)^2)$	$O(l \cdot n \cdot k^2)$
Aguilar-Melamine and Gaborit [1]	$O(l + n \cdot k)$	$O(l \cdot n \cdot \log(k) \cdot \log(\log(k)))$
PIRMAP	$O(l + n \cdot k)$	$O(l \cdot n \cdot \log(k) \cdot \log(\log(k)))$

and Ostrovsky [12]. They also achieve much faster query response, but again, with large communication costs.

Comparison of existing work To understand how existing work compares and is suited to the application targeted in this paper, we start by examining asymptotic computation and communication complexities in Table 1. We can see that Lipmaa [13] has very good communication complexity with relatively bad computational complexity, while Aguilar-Melamine and Gaborit [1] have the opposite. While our protocol PIRMAP has the same asymptotic complexities than Aguilar-Melamine and Gaborit [1], this does not adequately describe the costs involved with PIR: we are primarily addressing a concrete, practical setting where constants become very important.

Consequently, in Table 2, we present analytically calculated communication costs for specific database and file sizes. Aguilar-Melamine and Gaborit [1] state that their requests are of size $25kb \cdot n$ and the responses are $6 \cdot l$. For databases with $l > n \cdot k$, Lipmaa [13] requires transmitting $\log^2(n)$ elements of size k in the request and receiving a response of size $\log(n) \cdot l$. We chose to use $k = 2048$, as suggested by Lipmaa [13].

In Table 2, we can see that even though Lipmaa [13] has good asymptotic communication, there is room for improvement in a concrete setting, especially for databases composed of large files. Aguilar-Melamine and Gaborit [1], similarly to PIRMAP, is designed specifically for these types of databases but it also has relatively high communication costs due to large constants.

Our approach In conclusion, we would like to achieve, in real world settings, the small communication cost of Lipmaa [13] with the fast query response times of Aguilar-Melamine and Gaborit [1] and Trostle and Parrish [20]. Lipmaa notes that, when $\alpha = 1$, the PIR scheme is particularly efficient for databases where $l > n \cdot k$. We choose this case as a base for our new protocol PIRMAP, because it requires only one iteration of the database during **Transfer** and has communication complexity of $O(l + n \cdot k)$. When $l > n \cdot k$, this is optimal, since the server response must always be of size l . In short, PIRMAP can be thought of as a combination of this protocol with the fast encryption scheme used by Trostle and Parrish [20]. This new “somewhat” homomorphic encryption scheme requires only modular multiplications rather than exponentiations, significantly improving the asymptotic and real world computational requirements.

Table 2: Real communication costs of related work in MB for different database sizes.

	1 MB Files			5 MB Files			10 MB Files		
	1 GB	10 GB	100 GB	1 GB	10 GB	100 GB	1 GB	10 GB	100 GB
Lipmaa	10	13	17	38	55	71	66	100	133
Aguilar-Gaborit	29	240	2350	35	77	499	62	83	295
PIRMAP	3	6	38	11	12	18	22	22	26

4 PIRMAP

PIRMAP modifies the PIR protocol by Lipmaa [13], specifically addressing the shortcomings perceived in regards to retrieval of large files in a parallelization-aggregation computation framework such as MapReduce. We start by giving an overview of PIRMAP which can be used with *any* additively homomorphic encryption scheme.

Upload. In the following, we assume that the cloud user has already uploaded its files into the cloud using the interface provided by the cloud provider.

Query. Our data set holds n files, each of l bits length. Additional parameter k specifies the block size of a cipher. For ease of presentation, we consider the case where all files are the same length, but PIRMAP can easily be extended to accommodate variable length files using padding. PIRMAP is summarized as follows:

1. **Query** – If the user wishes to retrieve file $1 \leq x \leq n$ out of the n files, he creates a vector $\mathbf{v} = (v_1, \dots, v_n)$, where $v_x = \mathcal{E}(1)$ and $v_i = \mathcal{E}(0) \forall i \neq x$. Here, \mathcal{E} is any IND-CPA additively homomorphic encryption scheme. The user sends \mathbf{v} to the cloud.
2. **Multiply** – The cloud arranges files into a table \mathcal{T} similar to Table 3. The cloud divides each file i into $\frac{l}{k}$ blocks $\{B_{i,1}, \dots, B_{i,\frac{l}{k}}\}$ and multiplies each block by v_i , i.e., $B'_{i,j} = v_i \cdot B_{i,j}$. Here, “ \cdot ” denotes scalar multiplication.
3. **Sum** – The cloud adds column-wise to create one result vector $\mathbf{r} = (r_1, \dots, r_{\frac{l}{k}})$. Each element r_i of that vector is of size k , so the total bit length of \mathbf{r} is l bits. Vector \mathbf{r} is an encryption of file x . Vector \mathbf{r} is returned to the user who decrypts it.

Table 3: Cloud splits files into pieces

	1	2	\dots	$\frac{l}{k}$
$file_1$	$B_{1,1}$	$B_{1,2}$	\dots	$B_{1,\frac{l}{k}}$
$file_2$	$B_{2,1}$	$B_{2,2}$	\dots	$B_{2,\frac{l}{k}}$
\dots			\dots	
$file_n$	$B_{n,1}$	$B_{n,2}$	\dots	$B_{n,\frac{l}{k}}$

The cloud performs the matrix-vector multiplication $\mathbf{r} = \mathbf{v} \cdot \mathcal{T}$. In practice, the cloud does not need to create a table, but just needs to perform the block wise scalar multiplications and additions.

The computation consists of multiplying each of the $n \cdot \frac{l}{k}$ blocks by a value in the request vector and then performing $(n - 1) \cdot \frac{l}{k}$ additions to obtain the final sum. The computational complexity of this scheme is $O(n \cdot l \cdot M(k) + (n - 1) \cdot l \cdot A(k))$, where $M(k)$ is the cost of performing one scalar multiplication in the additively homomorphic encryption system, and $A(k)$ is the cost of one addition. The communication complexity can be broken down into two parts: user to cloud and cloud to user. The user sends a vector of size n containing ciphertexts of size k , so the bandwidth complexity user-cloud is $O(n \cdot k)$. The cloud sends back a vector of $\frac{l}{k}$ entries, each of size $O(k)$, resulting in complexity $O(l)$. This results in the overall communication complexity of $O(n \cdot k + l)$.

PIRMAP achieves better overhead than related work [4, 13] in practice, due to the specific values of n and l in our setting. Existing work in PIR is efficient *only* for data sets with large values for n and small values for l . PIRMAP would do poorly in that setting, because the complexity would be dominated by n . However, PIRMAP’s cloud-to-user communication is *optimal* at $O(l)$, because the cloud must send back a message at least the size of the file the user queries for. For values of l larger than $n \cdot k$, PIRMAP allows for complexity of $O(l)$ with small constants. We argue that in practice this is often true. For example, if a user has a 1 TB data set of 10 MB files, $\frac{l}{n} \approx 800$. In contrast, under the same conditions, arranging the files in a $\sqrt{n} \times \sqrt{n}$ matrix would result in a download cost of $1024 \cdot 10 \text{ MB} \approx 10 \text{ GB}$. Even if $n > l$, the actual communication costs are low for practical choices of the parameters, see Section 5. Many cloud providers such as Amazon do not charge the user for uploads, but only for downloads. This is an additional benefit of our scheme, because the query result (communication the user is charged for) is always very close to optimal, even when the overall communication is not.

Our assumption of $l > n$ is realistic and useful for many real-world applications. For example, medical records may contain one or more images (x-ray, MRI, etc) which would make them several megabytes at the very least.

Optimization: Although the cloud performs most of the computation in this scheme, the user is still required to generate a vector of ciphertexts of length n and then decrypt the resulting response. As encryption is relatively expensive for additive IND-CPA ciphers, this might require a non-trivial amount of computation that might hurt, e.g., users with low-powered devices such as smartphones. A way to alleviate this problem is to have a moderately powerful trusted server to pre-generate vectors of ciphertexts and upload them to the cloud for the users to use. This trusted server would generate m “disposable” vectors of size n such that $V_{i,j} = \mathcal{E}(1)$ where $j = \text{HMAC}(i)$ and $V_{i,j} = \mathcal{E}(0)$ otherwise. This allows the user to use one of these disposable vectors at query time and permute it so that the single $\mathcal{E}(1)$ is at the index of the file it wishes to retrieve. If the key used in the HMAC is shared between the user and trusted server, the user can efficiently locate $\mathcal{E}(1)$. The user then generates a description of a permutation which moves the $\mathcal{E}(1)$ value to the correct position and randomly shuffles all other locations. A description of this permutation is of size $n \cdot \log(n)$ which is smaller than the size of the vector for $k > 30$. This also effectively front loads the upload cost of the query and makes response time even faster.

Although the particular encryption scheme we use (see Section 4.2 below) can perform encryptions very quickly and does not require the use of this optimization, we point it out as a general improvement regardless of the cipher used.

4.1 PIRMAP Specification

In our protocol, the cloud performs two operations: multiplication of each block by the corresponding value in the “PIR vector” v , and column-wise addition to construct the encrypted file chosen by the user. These two stages translate exactly to map and reduce implementations respectively. The files will be distributed evenly over all participating nodes where the map function will split each file into blocks and multiply the blocks by the correct encrypted value. The output of these mappers is a set of key-value pairs where the key is the index of the block, and the value is the product of the block and encrypted PIR value. These values are all passed on to the reducers, which take a set of values for each key (block position or column) and add them together to get the final value for each block.

Being interested in $file_x$, the user executes the above **Query** to compute v which is sent to the MapReduce cloud. There, each mapper node evaluates **Multiply** on its locally stored file and generates key-values pairs for the reducer. The reducer simply computes the **Sum** step by adding all values with the same key and sending them back to the user. The user receives $\frac{l}{k}$ values of size k from the reducers and decrypts to get $file_x$.

User	Cloud
<pre> function GenQuery(n, x) $v := \{\}$ for $i = 1$ to n do if $i = x$ then $v_i := \mathcal{E}(1)$ else $v_i := \mathcal{E}(0)$ end if end for end function </pre>	<pre> function Map($file, v$) for $i = 1$ to n do $c := B_i \cdot v_i$ Emit(i, c) end for end function function Reduce(key, v) $total := 0$ for $i = 1$ to n do $total := total + v_i$ end for Emit($key, total$) end function </pre>

4.2 Encryption Scheme

Since the map phase of our protocol involves multiplying every piece of the data set by an encrypted PIR value, it is important that we choose an efficient cryptosystem. Traditional additively homomorphic cryptosystems, such as Paillier’s, use some form of multiplication as their homomorphism. That is, for elements a and b , $\mathcal{E}(a) \cdot \mathcal{E}(b) = \mathcal{E}(a + b)$. Since our map phase consists of multiplying ciphertexts by unencrypted scalars, we would have to do exponentiation of a ciphertext. PIRMAP, and all PIR schemes, must compute on the whole data set, so this step would be computationally intensive.

Similar to our recent findings [3], we mitigate this problem by using a *somewhat homomorphic* encryption scheme introduced by Trostle and Parrish [20] that relies on the

trapdoor group assumption. Fully homomorphic encryption schemes support an unlimited number of computations without increasing the size of the ciphertexts. In contrast, this scheme results in ciphertexts which grow in size by $O(\log_2 n)$ bits for n additions. In return for this size increase, we can have an encryption scheme where the additive homomorphism is integer addition. This scheme encrypts n bits with security parameter $k > n$ as follows:

KeyGen(1^k): Generate a prime m of k bits and a random $b < m$; b and m are secret.
Encrypt $\mathcal{E}(x) = b \cdot (r \cdot 2^n + x) \pmod m$, for a random r
Decrypt $\mathcal{D}(c) = b^{-1} \cdot c \pmod m \pmod{2^n}$

This encryption has the desired homomorphic property $\mathcal{E}(a) + \mathcal{E}(b) = \mathcal{E}(a + b)$. This scheme is *somewhat homomorphic*, because it cannot support an unlimited number of additions. When two ciphertexts c_1 and c_2 are added, you can express the sum as

$$b \cdot (r_1 \cdot 2^n + x_1) + b \cdot (r_2 \cdot 2^n + x_2) = b \cdot ((r_1 + r_2) \cdot 2^n + x_1 + x_2).$$

As m remains secret, note that the cloud performs addition as integer addition, not modulo m . If the inside term $(r_1 + r_2) \cdot 2^n + x_1 + x_2$ exceeds m and “wraps around”, then it will not be decrypted correctly, because application of the modulus will cause a loss of information. The modulus m must be chosen large enough to support the number of additions expected to occur. To support t additions, m should be increased by $\log_2 t$ bits. Additionally, each scalar multiplication can be thought of as up to 2^n additions, meaning that the size of m must be doubled for each supported scalar product. For our PIR scheme, m must be chosen to be $O(2k + \log_2(n))$ to support the required homomorphic operations.

In return for the reasonable increase in ciphertext size caused by the larger modulus (about 300% in our evaluations in Section 5), we are able to do very efficient computations over the encrypted data. Additionally, encryption is equivalent to only two multiplications, an addition and a modular reduction, while decryption is one multiplication and a reduction. This compares very favorably with other homomorphic encryption schemes, such as Paillier, requiring a modular exponentiation.

With our encryption scheme defined, we can now express more precise computational complexities for the protocol. Our previous complexity was parameterized over $M(k)$ and $A(k)$, the complexity of scalar multiplication and addition, respectively. For our encryption, addition is simply regular integer addition. Since each cipher text is at most $2k + \log_2(n)$ bits long, addition is $O(2k + \log_2(n))$. We can do scalar multiplication as integer multiplication as well. Integer multiplication can be done for m bits in $O(m \log(m) \log(\log(m)))$ [9], so $M(k) = O(2k + \log_2(n) \log(2k + \log_2(n)) \log(\log(2k + \log_2(n))))$. The complexity is then dominated by the multiplication cost and results in:

$$O\left(n \cdot \frac{l}{k} \cdot (k + \log(n)) \log(k + \log(n)) \log(\log(k + \log(n)))\right) \quad (1)$$

$$= O\left((n \cdot l + n \cdot \log(n)) \log(k + \log(n)) \log(\log(k + \log(n)))\right) \quad (2)$$

If $l > n$, then $l > \log(n)$, and we can simplify to $O(n \cdot l \cdot \log(k + \log(n)) \cdot \log(\log(k + \log(n))))$. Additionally, k has to be much larger than $\log(n)$, otherwise the

server has the resources to find key (m, k) by brute force. This allows us to simplify to $O(n \cdot l \cdot \log(k) \cdot \log(\log(k)))$.

4.3 Privacy Analysis

PIRMAP inherits privacy properties of the work it is based on, i.e., Lipmaa [13] and the PIR variant by Trostle and Parrish [20]. In the following, we sketch our privacy rationale.

PIRMAP is privacy-preserving, **iff** an adversary (the cloud) cannot guess, after each query, with probability greater than $1/n$, which file was retrieved by the user after an invocation of the protocol. There are two pieces of information that the adversary has access to: the set of uploaded files and the vector v of PIR values. The uploaded files are independent of any encryption used in the PIR protocol and can be efficiently simulated by the adversary. Therefore, privacy depends only on v .

Vector v contains many encryptions of “0” and one encryption of “1”. The problem of determining which file was selected is then equivalent to *distinguishing* between encryptions of “0” and encryptions of “1” in the underlying encryption. However, the scheme we use is provably secure against distinguishing under the Trapdoor Group Assumption [20]. Consequently, PIRMAP preserves user privacy.

5 Evaluation

We have evaluated the performance of our scheme in three contexts: a local “cloud” (a single server with multiple CPUs), a commodity laptop, and Amazon’s EC2 cloud using Elastic MapReduce [2]. We have implemented PIRMAP in Java for standard Hadoop MapReduce version 1.0.3 and the source code is available for download [17].

5.1 Setup

Local We used a local server to prototype and debug our application and to do detailed timing analysis requiring many runs of MapReduce. This server, running Arch Linux 2011.08.19, has a dual 2.4 GHz quad-core Xeon E-5620 processor and 48 GB of memory. Based on specs and benchmark results, this local server is closest to an “EC2 Quadruple Extra Large” instance, which has dual 2.9 GHz quad-core Xeon X-5570 processors and 24 GB of memory.

We have measured the time needed for PIR queries, i.e., the time to upload PIR vector v plus the time to process the query and download the result. Using Amazon’s standard cost model, we have calculated the price of each PIR query as the amount of money required to run the query on one of the above EC2 instances (for the same amount of time it took to run locally) plus the bandwidth cost of downloading the results [2]. Since uploading data to Amazon is free, this does not add any additional cost. To put our measurements into perspective, we have also evaluated time and cost of two other, hypothetical, PIR protocols. We have implemented a *Baseline*, which does not perform *any* cryptographic operations and merely “touches” each piece of data through the MapReduce API. This measure shows the theoretical *lower bound* of computation

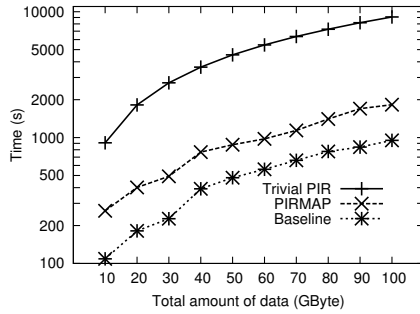


Fig. 1: Time per query, local server

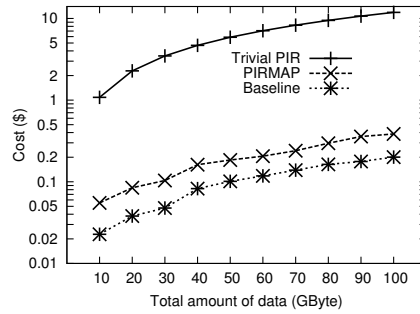


Fig. 2: Cost per query, local server

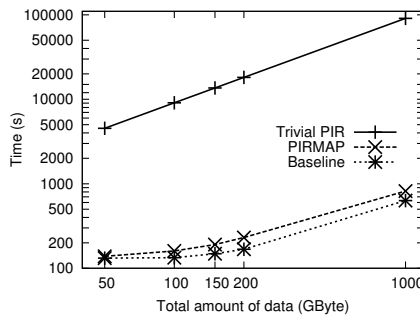


Fig. 3: Time per query, Amazon

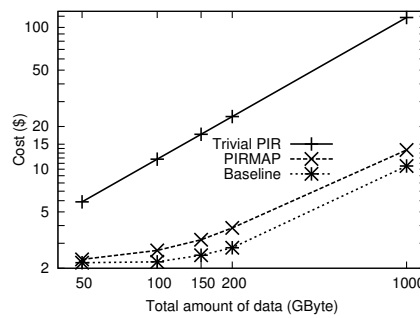


Fig. 4: Cost per query, Amazon

and time required for *any* PIR scheme that uses MapReduce, independent of the encryption and exact PIR method used.

To highlight the advantage of computational PIR, we have also included the time and cost required for the “trivial” PIR scheme. The trivial scheme is one where the user downloads the entire data set and simply discards the files he is not interested in. This is very bandwidth intensive, but computationally lightweight. Sion and Carbunar [18] conjecture this trivial PIR to be the most cost effective in the real-world. We have calculated the cost based on the amount that Amazon charges to download the corresponding amount of data and the time based on a 11.28 Mbps connection, an average as reported by Nasuni [15]. Note that we generally do not count the cost for long-term storage of data at Amazon. Although potentially significant for large amounts of data, the user has to pay for this regardless of whether he wants queries to be privacy-preserving or not. PIRMAP does not increase the amount of storage in Amazon.

Laptop We also ran our implementation on a 2012 MacBook Pro with a 2.3 Ghz i7, 8 GB of RAM and an NVIDIA GeForce 650M. The purpose of this test was to compare query time with related work, and the results are shown in Table 4. The Aguilar scheme takes advantage of GPU resources, so we chose a machine with comparable graphics and GPU resources (unlike the above server, which had no discrete graphics). This represents the performance you would get on a commodity machine and also

shows that query generation is not very taxing. To compare with Lipmaa [13], we used `openssl speed rsa` to time determine how many RSA private key operations can be computed per second. Each scalar multiplication in that scheme is equivalent to one modular exponentiation, or one RSA private key operation as required by Lipmaa [13]. This estimation is quite generous for Lipmaa [13], because it does not include disk access or homomorphic additions (modular multiplication), but we believe it is close due to the time being largely dominated by exponentiations.

Table 4: Query time in minutes, including generation, evaluation, and decryption, for databases of varying sizes, composed of 5 MB files.

	5 GB	10 GB	15 GB	20 GB	25 GB	30 GB
Lipmaa	1852	3704	5508	7312	9116	10920
Aguilar-Gaborit	4	7	11	14	17	20
PIRMAP	1	2	3	3	4	4

Amazon Besides the local experiments, to demonstrate the scalability of PIRMAP, we have also evaluated it on Amazon’s Elastic MapReduce cloud. Amazon imposes a maximum limit of 20 instances per MapReduce job by default. In keeping with this restriction, we used 20 “Cluster Compute Eight Extra Large” instances which each having dual eight-core Xeon E5-2670 processors and 64 GB of RAM.

5.2 Results

Time and total cost: Figures 1 to 4 show our evaluation results. Figures 1 and 2 show the local server evaluation, while figures 3 and 4 show evaluation with Amazon. In each figure, the x-axis shows the total amount of data stored at the cloud, i.e., number of files n times file size l . The y-axis shows either the time elapsed (for the whole query from the time the user submits the query until MapReduce returns the result back) or the cost implied with the query. In all four graphs, we scale the y-axis logarithmically, and in figures 3 and 4 we also scale the x-axis logarithmically. Each data point represents the average of at least 3 runs. Relative standard deviation was low at $\approx 5\%$.

To verify the impact of varying file sizes l , for our local evaluation, we show results with file sizes of 1 MB and 3 MB, attaining approximately equal runtime in both cases. This is to be expected because, in each data point, we have fixed the size of the database so varying retrieval sizes merely reshapes the matrix without changing the number of elements in it. The execution is dominated by the scalar multiplications that occur during the map phase, and the same number of those is required no matter the dimension of the matrix.

Our evaluation shows that PIRMAP outperforms trivial PIR in both time and cost by one order of magnitude. We also compare performance to a baseline PIR protocol implemented in MapReduce where the mappers simply read and ignore any rows that are not requested and return the single row that was. Any MapReduce-based PIR protocol will require at least this amount of computation. Compared to this theoretical, yet

unrealistic optimum, PIRMAP introduces only 20% of overhead in the case of Amazon. Locally, we experience slightly larger overhead of 100%. This is because executing on Amazon has a much higher “administrative” cost due to the higher number of nodes and more distributed setting. These results indicate not only PIRMAP’s efficiency over Trivial PIR and Baseline, but also its real-world practicality: in a small database comprising 10,000 files of size 1 MB each (10 GByte), a user can retrieve a single record in ≈ 3 min for only $\approx \$0.03$. In a huge data set with 1,000,000 files, a single file can be retrieved in ≈ 13 min for $\approx \$14$. In scenarios where it is necessary to retrieve data in a fully privacy-preserving manner, we conjecture that this to be acceptable.

Although a comparison with related research is not straightforward (as PIRMAP targets a very special scenario), we put our results into perspective with those of Aguilar and Lipmaa. We show that, while Lipmaa’s scheme has very good communication complexity in all cases, it is completely impractical due to the enormous amount of computation needed to respond to queries. We also show that our scheme is comparable with that of Aguilar, in terms of computation, and beats it in communication cost by a significant margin.

Query Generation and Decryption: Due to the efficiency of the encryption in PIRMAP, PIR query generation is very fast. One ciphertext (element of v) is generated for each file in the cloud, so the generation time is directly proportional to the number of files. We omit in-depth analysis, but in our trials on our commodity laptop running on a single core it takes about 2.5 seconds per 100,000 files in the cloud. Decryption is slightly more expensive than encryption, but we still managed, on the same machine, to decrypt approximately 3 MB per second. We conclude this overhead to be feasible for the real-world.

Bandwidth: PIRMAP introduces bandwidth overhead, 1.) to *upload* v , and 2.) to *download* the encrypted version of the file. For security, we set $k = 2048$ bit, so each of the n elements of vector v has size 2048 bit. For a data set with 10,000 files (10 GByte), this requires the user to upload ≈ 2.5 MByte. As this can become significant with larger number of files, we suggest to then use the optimization in Section 4, especially for constrained devices.

6 Conclusion

Retrieval of previously outsourced data in a privacy-preserving manner is an important requirement in the face of an untrusted cloud provider. PIRMAP is the first practical PIR mechanism suited to real-world cloud computing. In the case where a cloud user wishes to privately retrieve large files from an untrusted cloud, PIRMAP is communication efficient. Designed for prominent MapReduce clouds, it leverages their parallelism and aggregation phases for maximum performance. Our analysis shows that PIRMAP is an order of magnitude more efficient than trivial PIR and introduces acceptable overhead over non-privacy-preserving data retrieval. Additionally, we have shown that our scheme can scale to cloud stores of up to 1 TB on Amazon’s Elastic MapReduce.

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